La Struttura di Spin del Protone

M. Boglione

UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

INFN Torino
Istituto Nazionale di Fisica Nucleare

Working in collaboration with

M. Anselmino, A. Prokudin, A. Kotzinian, C. Türk
(Università di Torino e INFN Sezione di Torino)

U. D'Alesio, F. Murgia, S. Melis
(Università di Cagliari e INFN Sezione di Cagliari)

E. Leader
(Imperial College London)

Relevant experiments

HERMES @ DESY

COMPASS @ CERN

RHIC @ BNL

BELLE @ KEK

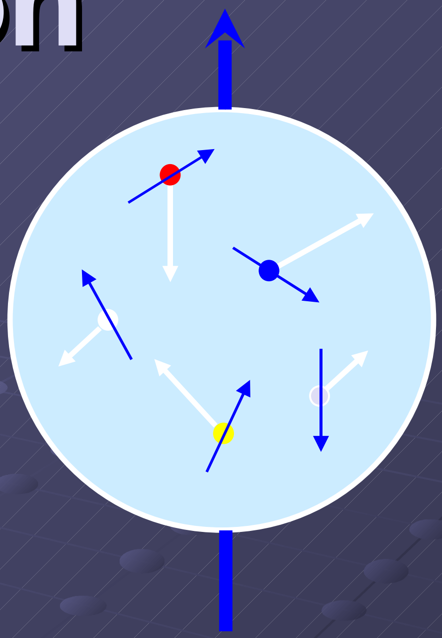
CLAS @ JLAB

(PAX @ GSI)

Summary

- Parton intrinsic motion: importance of $k_\perp$ in the study of the spin structure of the nucleons.
- Tranverse momentum dependent distribution and fragmentation functions.
- First determination of the transversity distribution function.
- Sivers mechanism.

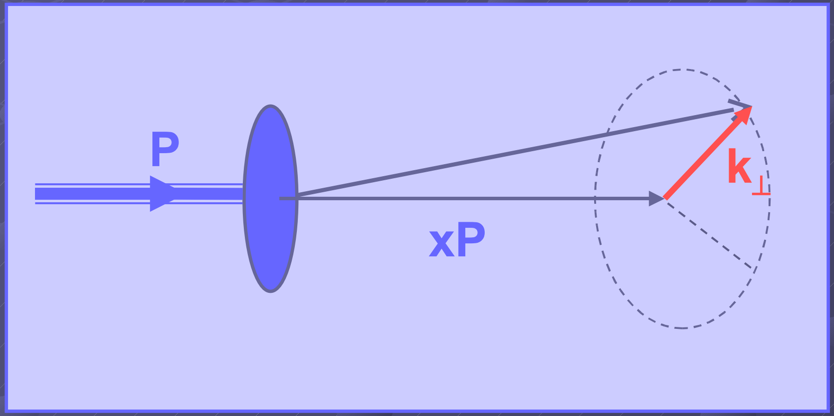
Partonic intrinsic motion



Plenty of theoretical and experimental evidence

for transverse motion of partons within nucleons

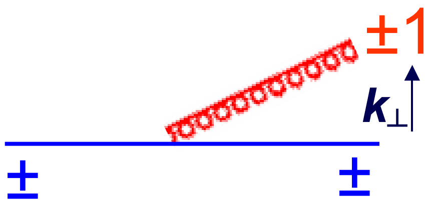
and of hadrons within fragmentation jets



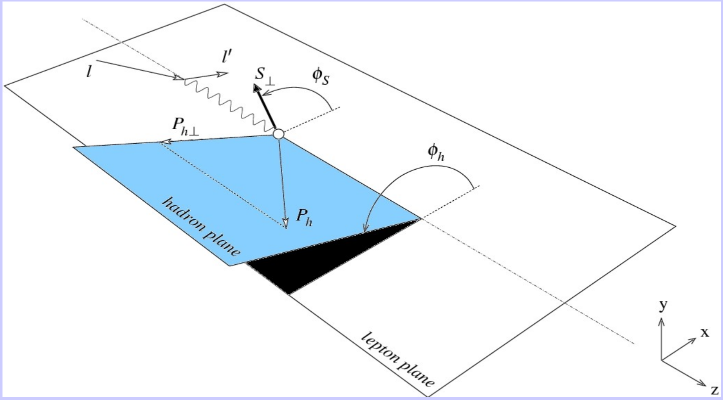
Uncertainty principle:

$\Delta x \sim 1$ fm => $\Delta p \sim 0.2$ GeV/c

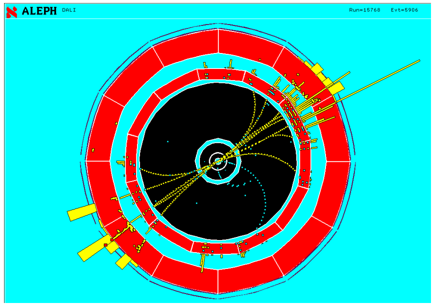
Gluon radiation



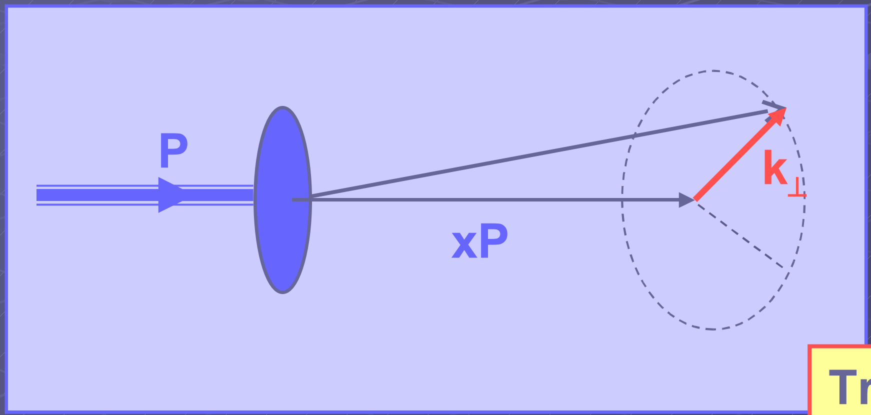
p_T distribution of hadrons in SIDIS

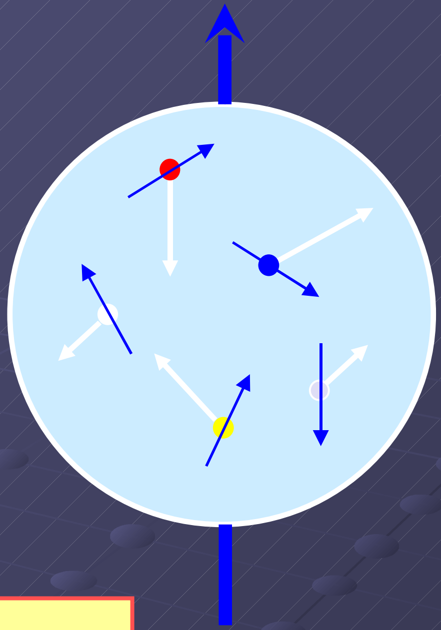


Hadron distribution in jets in e^+e^- processes



Partonic intrinsic motion





Transverse motion is usually integrated over, but there are important **spin-$k_\perp$** correlations !!

We cannot learn about the spin structure of the nucleon without taking into rigorous account the transverse motion of the partons inside it !!

Intrinsic motion in unpolarized SIDIS

$$d\sigma^{lp\to lhX}=\sum_q f_q(x,Q^2)\quad \mathrm{d}\,\{\hat{\sigma}^{lq\to lq}\quad D_q^h(z,Q^2)\text{¿}$$

in collinear parton model

$$x=\frac{Q^2}{2\mathrm{p}\cdot q}$$

$$Q^2=-q^2$$

$$y=\frac{p\cdot q}{l\cdot p}$$

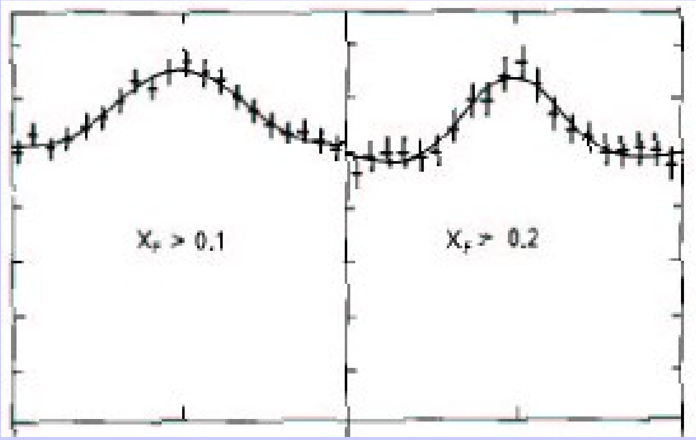
$$d\,\hat{\sigma}^{lq\to lq}\propto\hat{s}^2+\hat{u}^2\propto 1+(1-y)^2$$

thus, no dependence on azimuthal angle Φ_h at zero-th order in pQCD

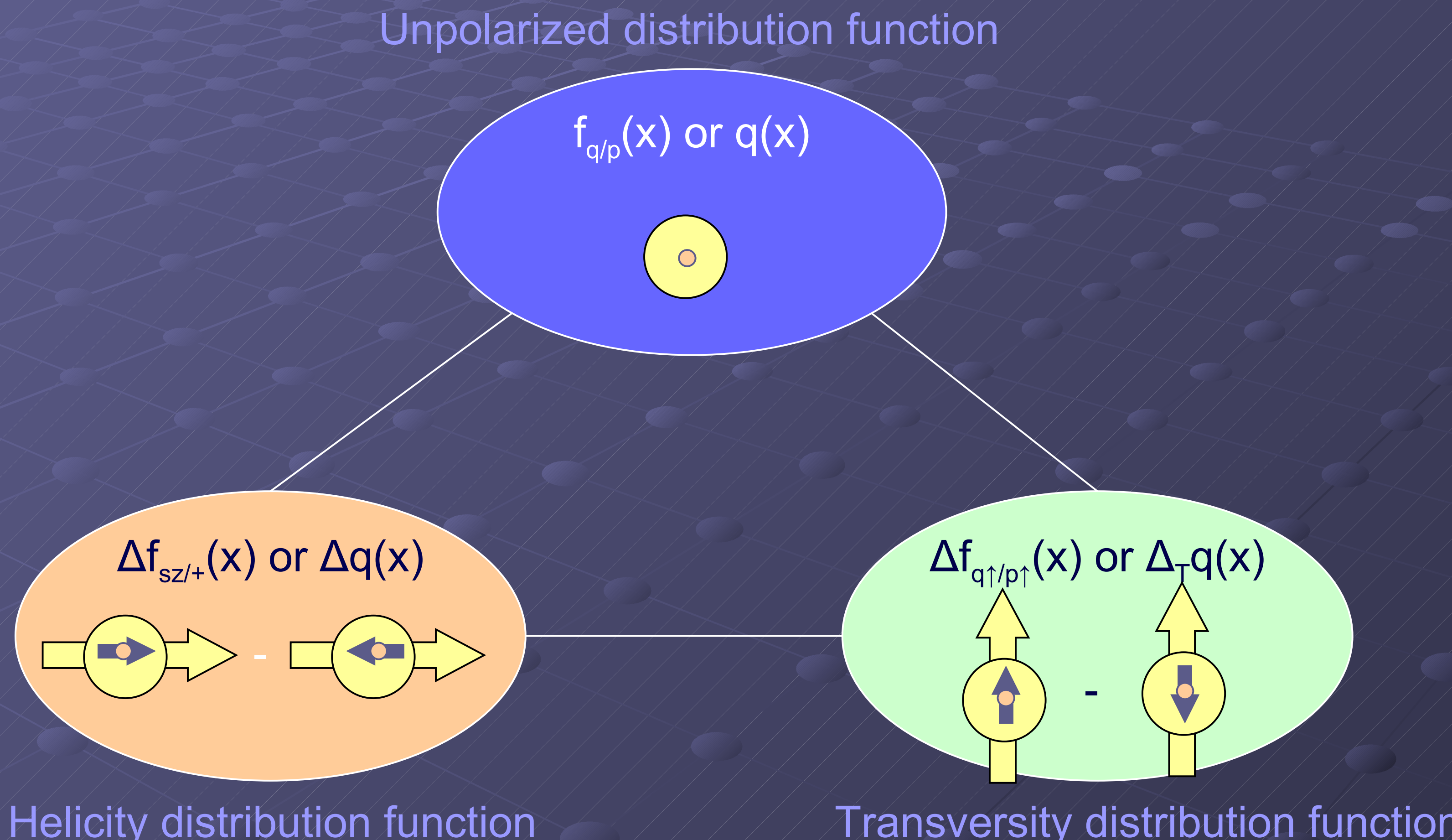
The experimental data reveal that

$$d\,\hat{\sigma}^{lq\to lh^{\pm}X}/d\Phi_h\text{¿}A+B\cos\Phi_h+C\cos 2\Phi_h$$

M. Arneodo et al (EMC): Z. Phys. C 34 (1987) 277

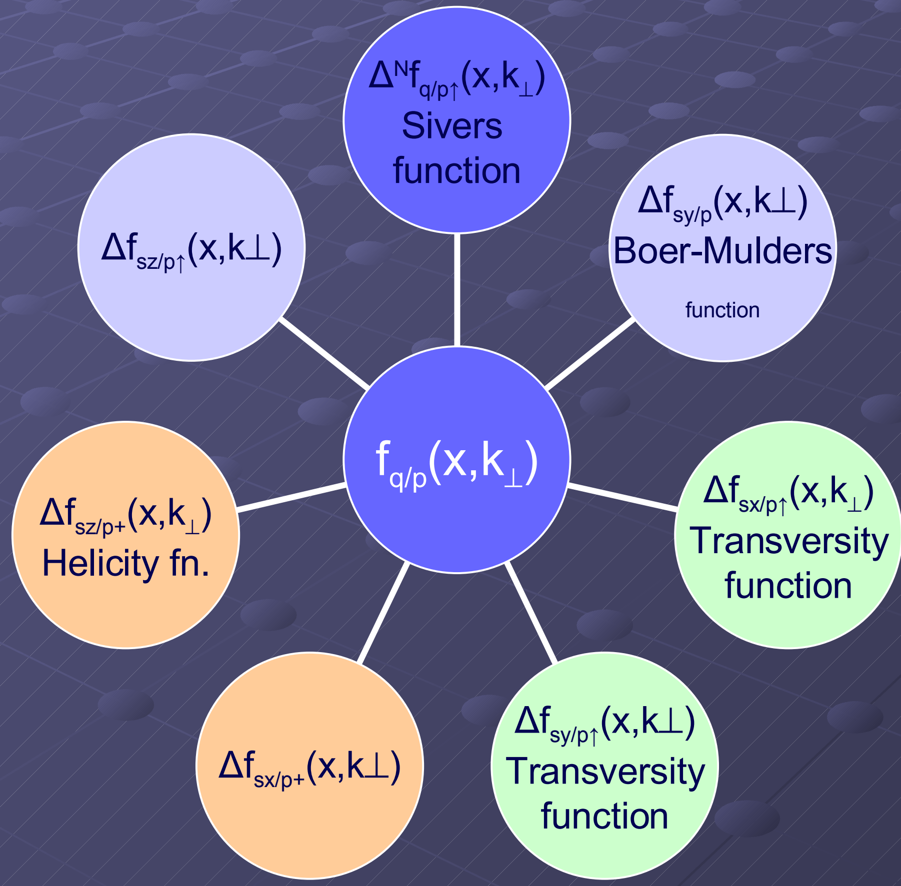


Integrated distribution functions

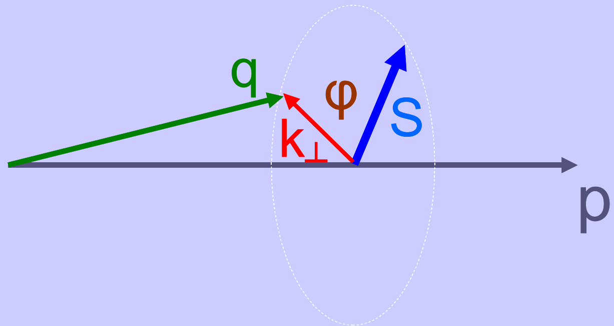


Transverse Momentum Dependent Distribution Functions

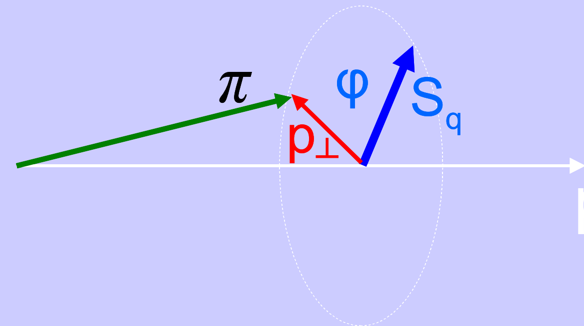
There are 8 leading-twist spin-$k_\perp$ dependent distribution functions



Spin - $k_\perp$ correlations



Sivers function



Collins function

$$f_{q/p^\uparrow}(x,\vec{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\vec{S}\cdot(\hat{p}\times\hat{k}_\perp)$$

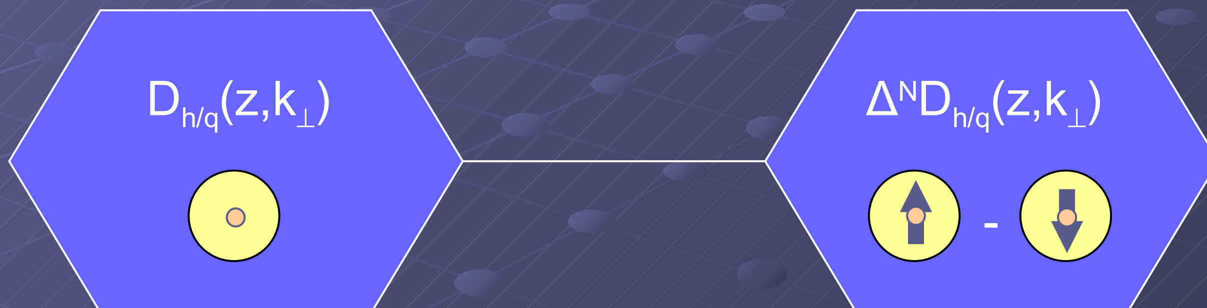
$$D_{h/q^\uparrow}(z,\vec{p}_\perp) = D_{h/q}(z,p_\perp) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z,p_\perp)\,\vec{S}_q\cdot(\hat{p}_q\times\hat{p}_\perp)$$

Amsterdam group notations

$$\Delta^N f_{q/p^\uparrow} = -\frac{2k_\perp}{M} f_{1T}^{\perp q}$$

$$\Delta^N D_{h/q^\uparrow} = 2\frac{p_\perp}{z M_h} H_1^{\perp q}$$

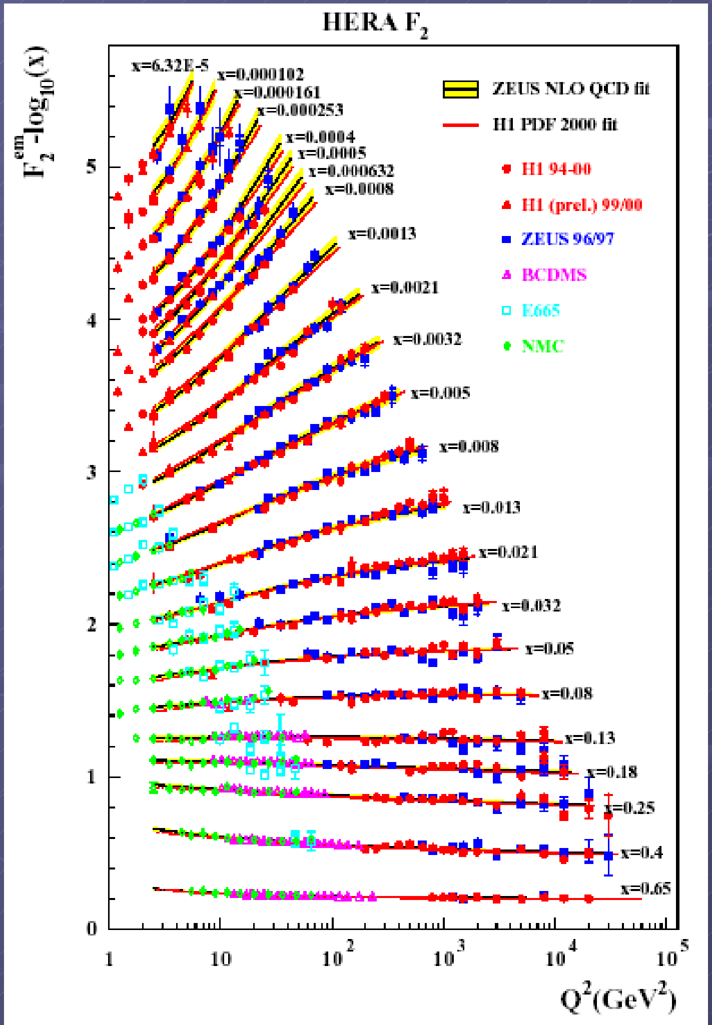
Transverse Momentum Dependent Fragmentation Functions

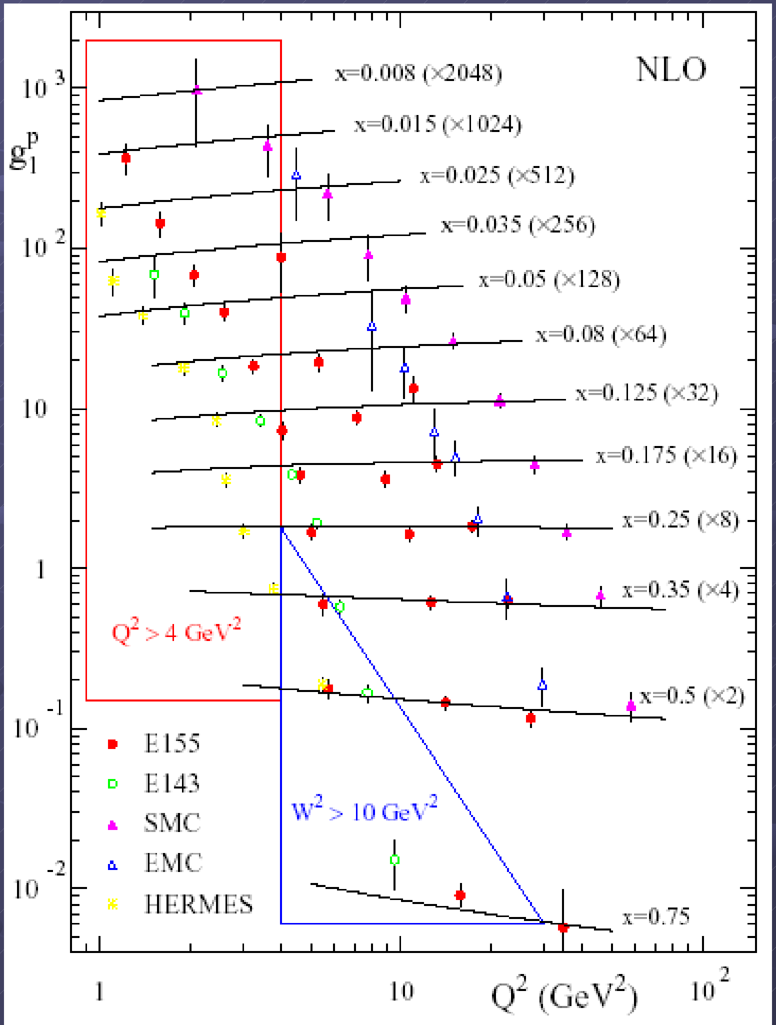


Unpolarized fragmentation function

Collins fragmentation function

Unpolarized and helicity distribution function are rather well known

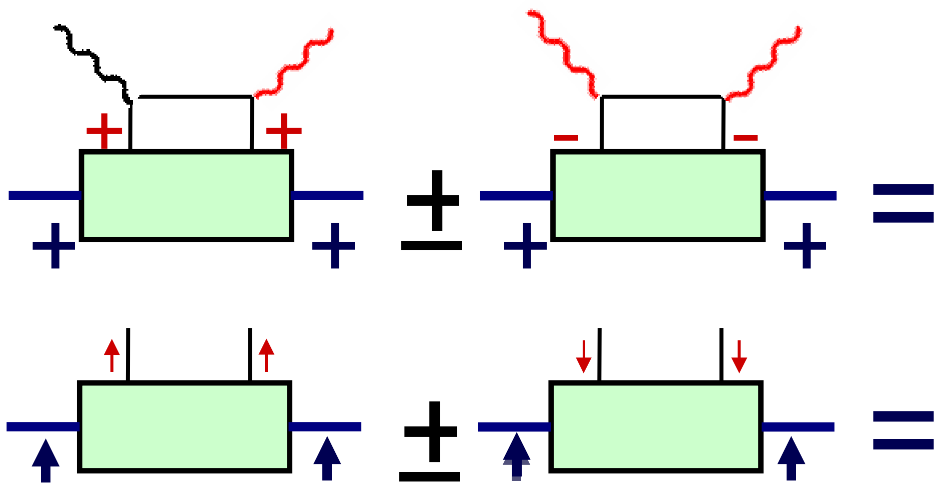




$\Rightarrow \quad q(x,Q^2)$

$\Rightarrow \quad \Delta q(x,Q^2)$

The missing piece of the puzzle: transversity

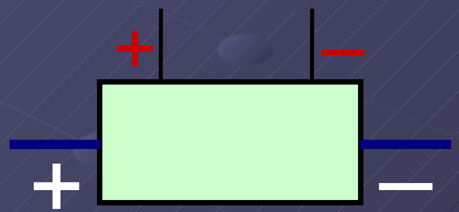


$$f_{q/p}(x,Q^2)$$
$$\Delta q(x,Q^2)$$

$$f_{q/p}(x,Q^2)$$
$$h_1(x,Q^2)$$

In helicity basis: ↑↓ $= \frac{1}{\sqrt{2}}\left(|+¿ \pm i|-¿\right)$

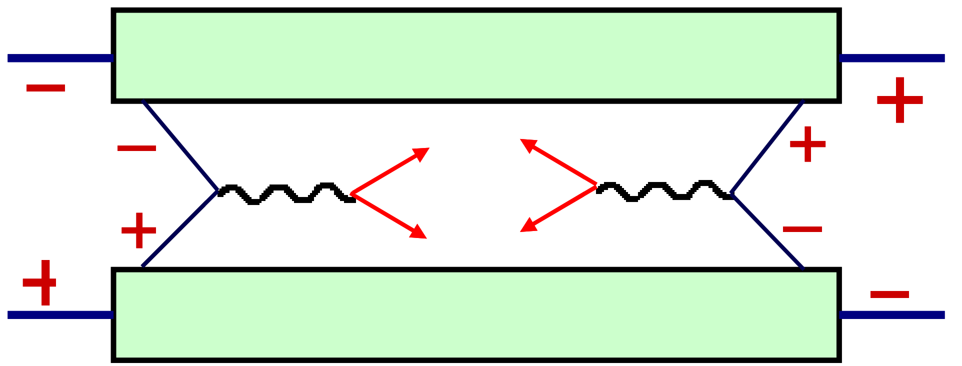
$$h_1(x,Q^2) =$$



decouples from DIS (no quark helicity flip)

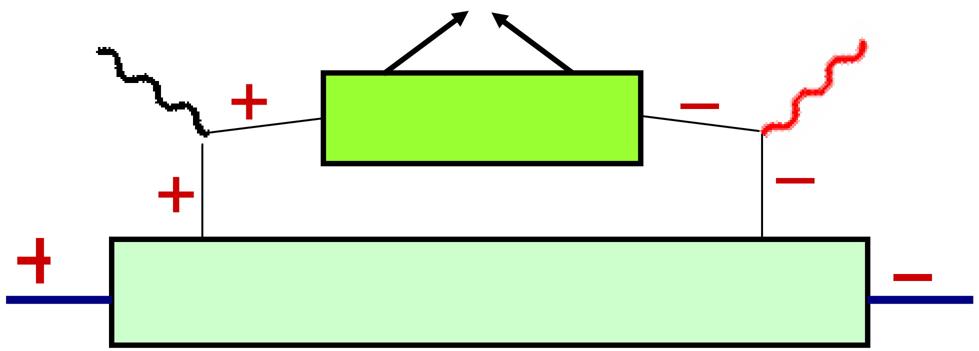
The transversity distribution function must couple to another chiral-odd function. For example

$$D-Y,\quad p\,p\to l^+\,l^-\,X\,,\ \text{and SIDIS,}\ l\,p\to l\,\pi\,X$$



h_1 x h_1

J. Ralston and D.Soper, 1979
J. Cortes, B. Pire, J. Ralston, 1992

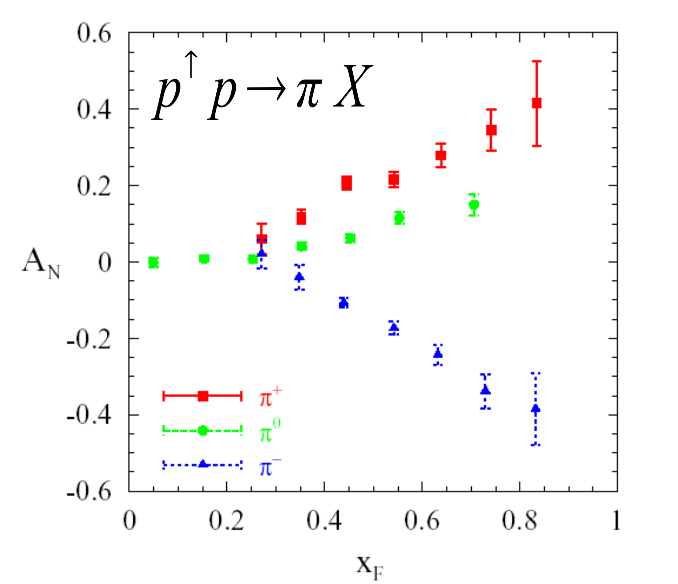


h_1 x $\Delta^N D$

J. Collins, 1993

Single spin asymmetries are excellent tools to study the spin structure of the nucleons, as they allow to disentangle different overlapping spin effects

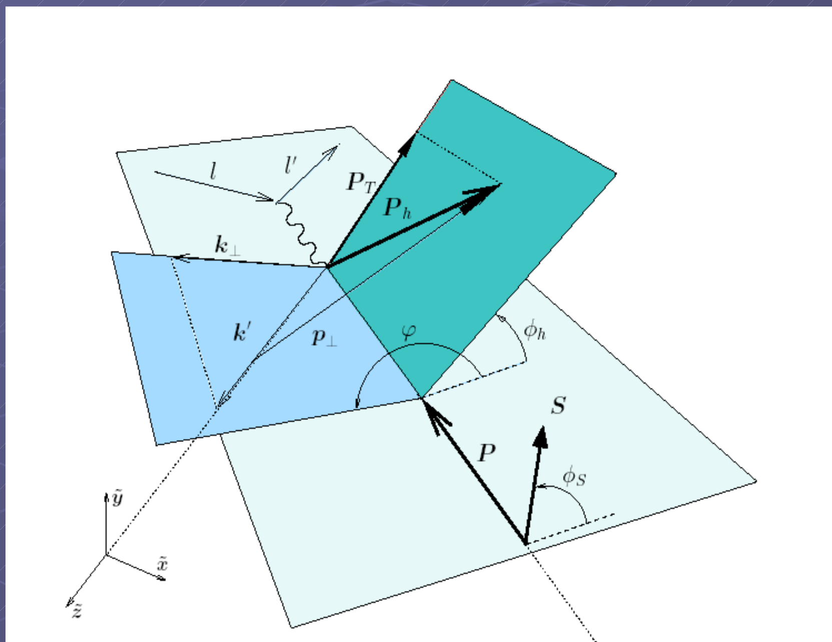
$$A = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



E704 $\sqrt{s}$ = 20 GeV
$0.7 < p_T < 2.0$

Transversity and Collins function from a global fit on SIDIS and e^+e^- data

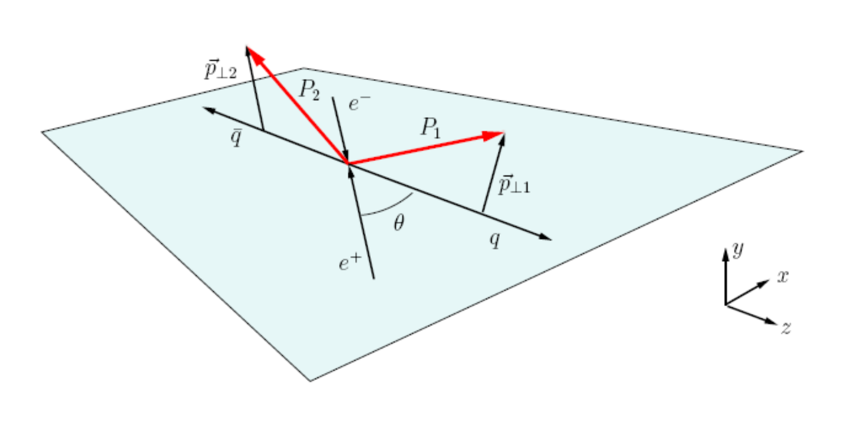
Transversity and Collins functions from SIDIS single spin asymmetry



$$A_{UT} = \frac{d\sigma^{\ell p^\uparrow \to \ell' hX} - d\sigma^{\ell p^\downarrow \to \ell' hX}}{d\sigma^{\ell p^\uparrow \to \ell' hX} + d\sigma^{\ell p^\downarrow \to \ell' hX}}$$

$$A_{UT}^{\sin(\varphi_S+\varphi_h)} = \frac{\sum_q e_q^2 \int d\varphi_S\, d\varphi_h\, d^2\vec{k}_\perp\, h_1(x,k_\perp)\, \frac{d(\Delta\sigma)}{dy}\, \Delta^N D_{h/q^\uparrow}(z,p_\perp)\, \sin(\varphi_S + \phi + \varphi_q^h)\, \sin(\varphi_S + \varphi_h)}{\sum_q e_q^2 \int d\varphi_S\, d\varphi_h\, d^2\vec{k}_\perp\, f_{q/p}(x,k_\perp)\, \frac{d\sigma}{dy}\, D_{h/q}(z,p_\perp)}$$

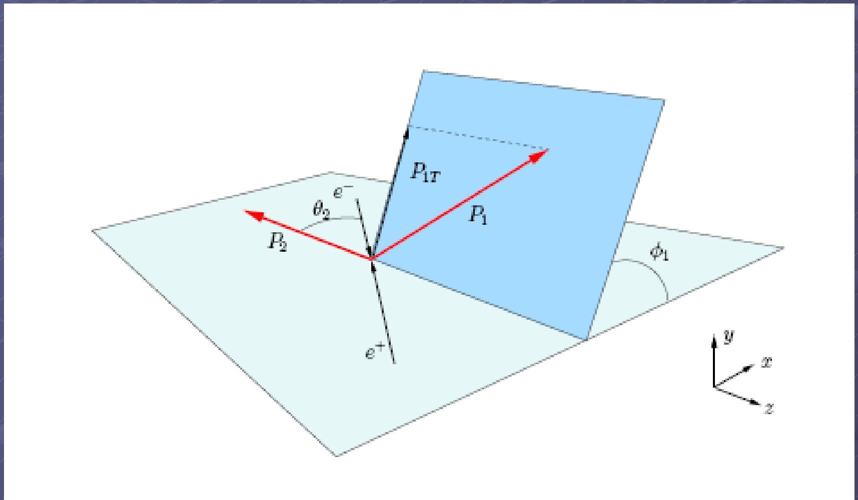
Collins function from $e^+e^- \to h_1 h_2\, X$ at BELLE



Thrust axis method

$$A(z_1, z_2, \theta, \varphi_1 + \varphi_2) \equiv \frac{1}{\langle d\sigma \rangle} \frac{d\sigma^{e^+e^- \to h_1 h_2 X}}{dz_1\, dz_2\, d\cos\theta\, d(\varphi_1 + \varphi_2)}$$
$$= 1 + \frac{1}{8} \frac{\sin^2\theta}{1 + \cos^2\theta} \cos(\varphi_1 + \varphi_2) \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

Collins function from e+e-→h1h2 X at BELLE



Hadronic plane method

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi}\,\frac{z_1\,z_2}{z_1^2 + z_2^2}\,\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,\cos(2\,\phi_1)\,\frac{\sum_q e_q^2\,\Delta^N D_{h_1/q^\uparrow}(z_1)\,\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\,D_{h_2/\bar{q}}(z_2)}$$

Parametrizations

Unpolarized distribution

Unpolarized fragmentation

$$f_{q/p}(x,k_\perp) = f_{q/p}(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle}\,,$$

$$D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}\,,$$

Gaussian widths

$$\langle k_\perp^2\rangle = 0.25\ \mathrm{GeV}^2 \qquad \langle p_\perp^2\rangle = 0.20\ \mathrm{GeV}^2$$

Transversity distribution

Collins fragmentation

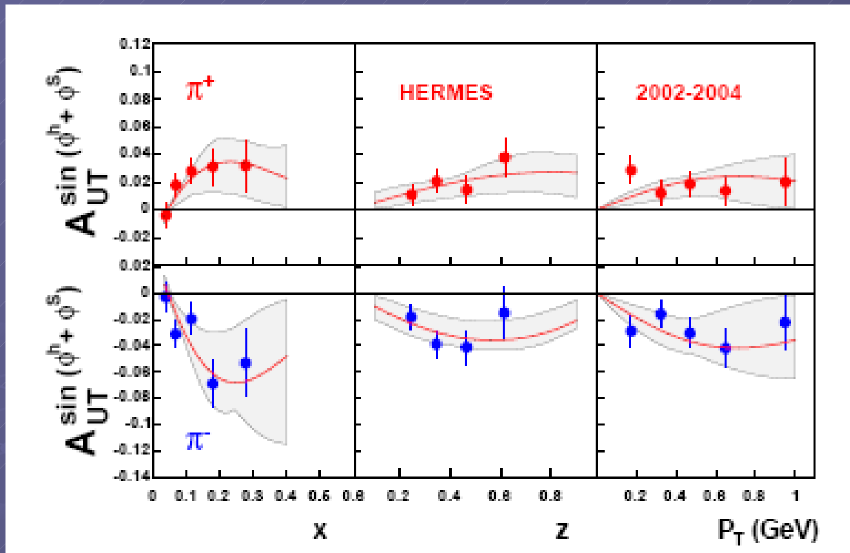
$$\Delta_T q(x,k_\perp) = \frac{1}{2}\mathcal{N}_q^T(x)\,\left[f_{q/p}(x) + \Delta q(x)\right]\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

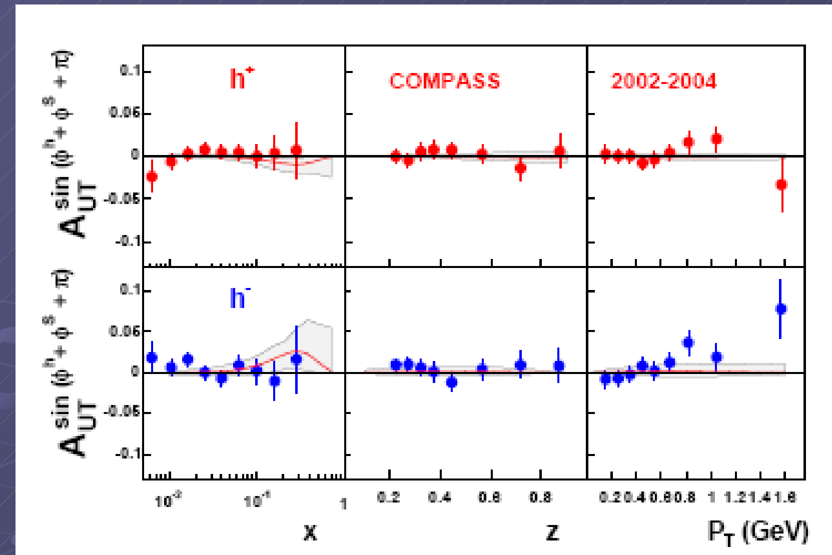
$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = 2\mathcal{N}_q^C(z)\,D_{h/q}(z)\,h(p_\perp)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}\,,$$

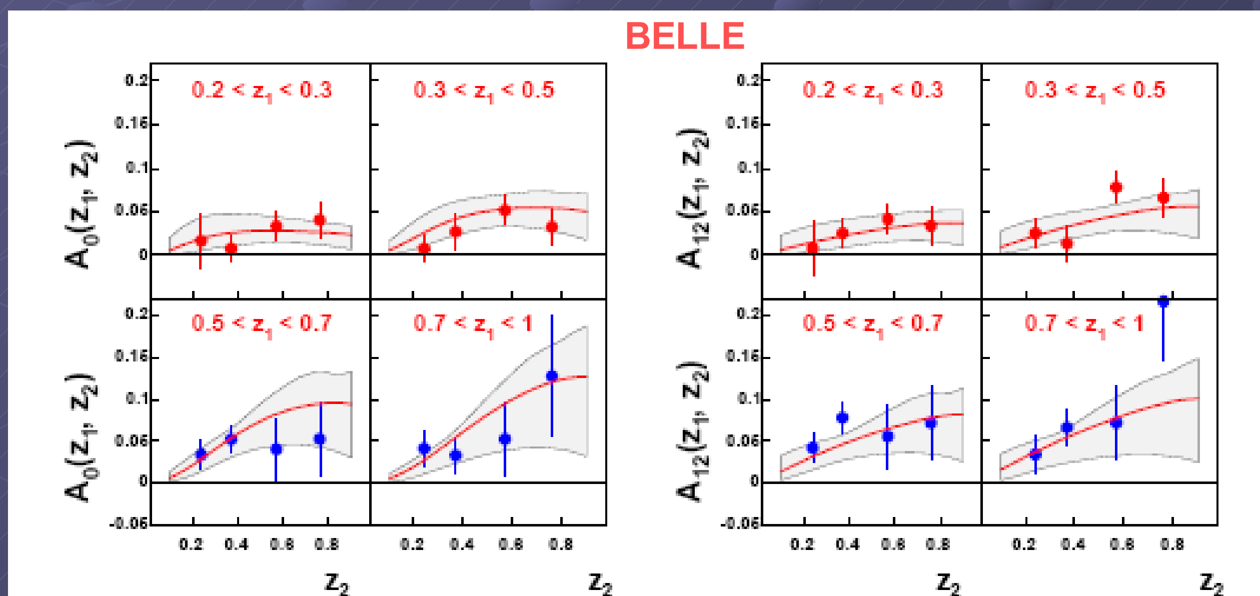
$$\mathcal{N}_q^T(x) = N_q^T\,x^\alpha(1-x)^\beta\,\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha\beta^\beta}$$

$$\mathcal{N}_q^C(z) = N_q^C\,z^\gamma(1-z)^\delta\,\frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma\delta^\delta}$$

$$h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M}\,e^{-p_\perp^2/M^2}\,,$$


π+
HERMES
2002-2004
π-
x
z
P_T (GeV)


h+
COMPASS
2002-2004
h-
x
z
P_T (GeV)


BELLE
0.2 < z1 < 0.3
0.3 < z1 < 0.5
0.5 < z1 < 0.7
0.7 < z1 < 1
A0(z1, z2)
A12(z1, z2)
z2

Collins fragmentation function

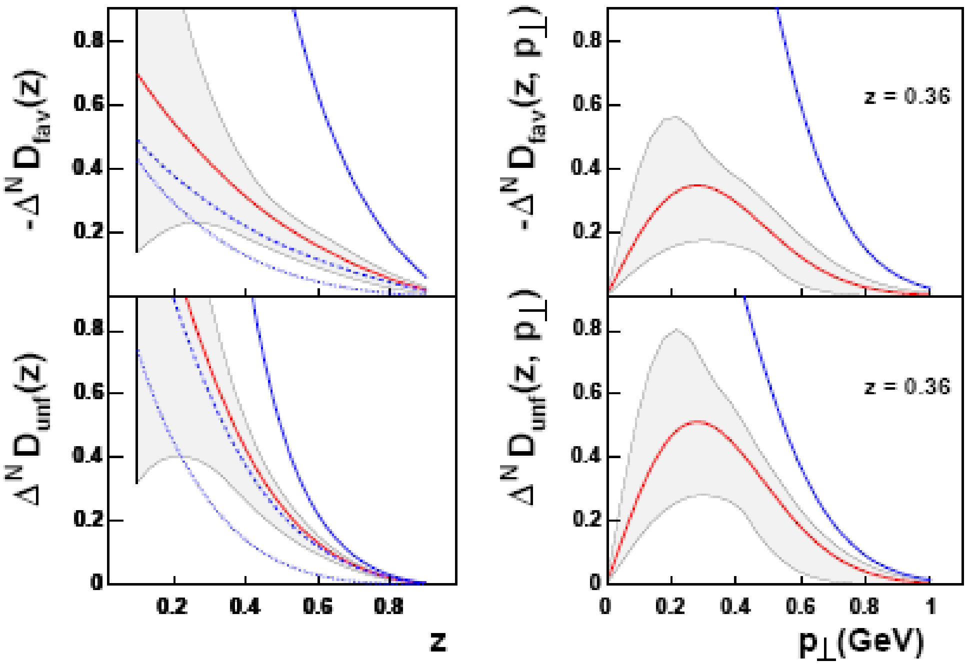


FIG. 8: Favored and unfavored Collins fragmentation functions as determined through our global best fit. In the left panel we show the z dependence of the $p_\perp$ integrated Collins functions defined in Eq. (40) and compare it to the results of Refs. [24] (dashed line) and [25] (dotted line). In the right panel we show the $p_\perp$ dependence of the Collins functions defined in Eq. (14), at a fixed value of z. In all cases we also show the positivity bound (19) (upper lines).

Transversity distribution function

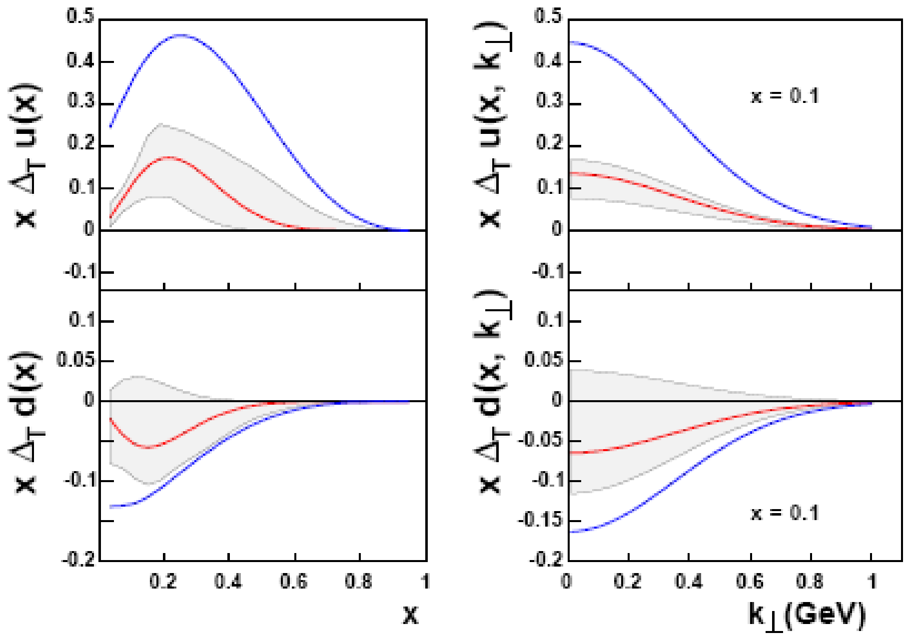
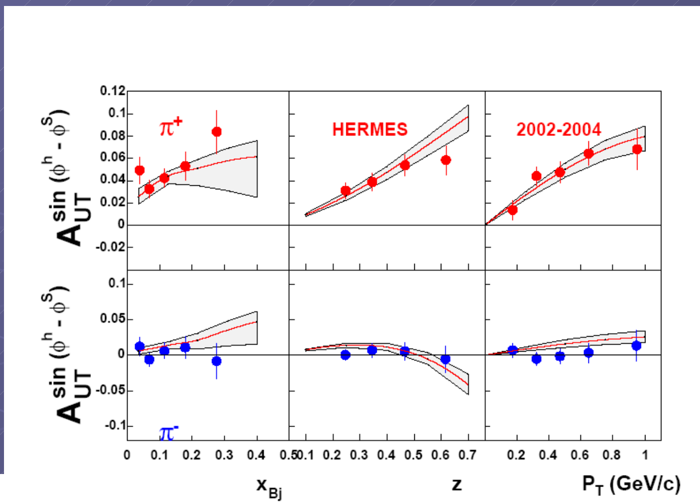
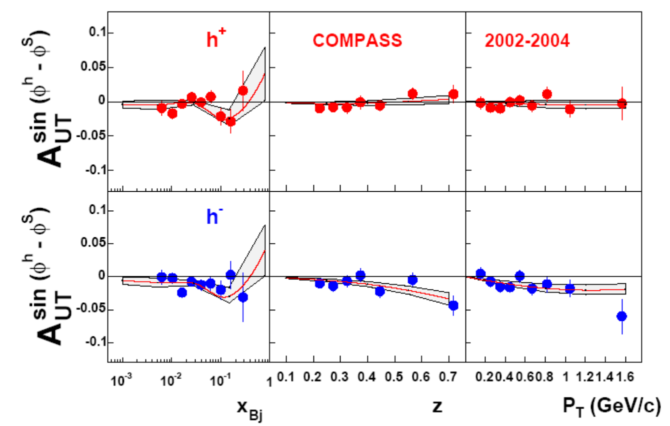
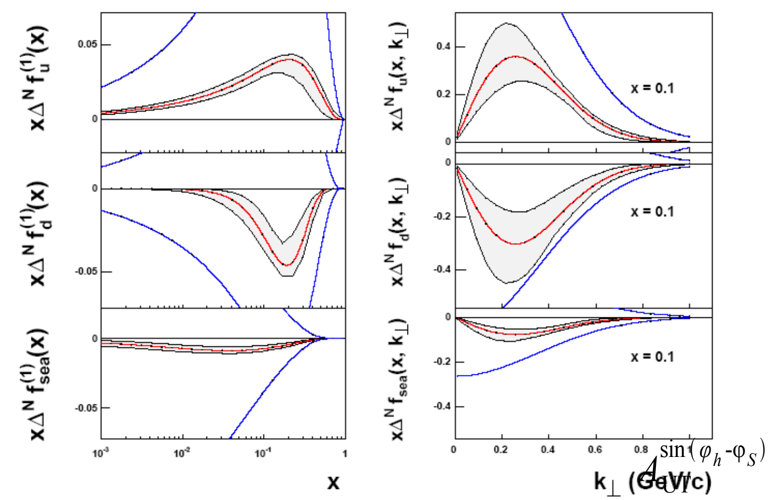


FIG. 7: The transversity distribution functions for u and d quarks as determined through our global best fit. In the left panel, $x\,\Delta_T u(x)$ (upper plot) and $x\,\Delta_T d(x)$ (lower plot), see Eq. (5), are shown as functions of x. The Soffer bound [20] is also shown for comparison (bold blue line). In the right panel we present the unintegrated transversity distributions, $x\,\Delta_T u(x, k_\perp)$ (upper plot) and $x\,\Delta_T d(x, k_\perp)$ (lower plot), as defined in Eq. (13), as functions of $k_\perp$ at a fixed value of x. Notice that this $k_\perp$ dependence is not obtained from the fit, but it has been chosen to be the same as that of the unpolarized distribution functions: we plot it in order to show its uncertainty (shaded area), due to the uncertainty in the determination of the free parameters.

Sivers distribution function from SIDIS data







$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = 2\mathcal{N}_q(x)\, h(k_\perp)\, f_{q/p}(x,k_\perp)$$

$$\mathcal{N}_q(x) = N_q\, x^{a_q}(1-x)^{b_q}\, \frac{(a_q+b_q)^{(a_q+b_q)}}{a_q^{a_q} b_q^{b_q}}$$

$-1 \le N_q \le 1$ $\quad |\mathcal{N}_q(x)| \le 1$

$$\frac{|\Delta^N f_{q/p^\uparrow}(x,k_\perp)|}{2 f_{q/p}(x,k_\perp)} \le 1$$

$$A_{UT}^{\sin(\varphi_h-\varphi_S)} = \frac{\sum_q e_q^2 \int d\varphi_S\, d\varphi_h\, d^2\vec{k}_\perp\; \Delta^N f_{q/p^\uparrow}(x,k_\perp)\, \frac{d\sigma}{dQ^2}\, D_{h/q}(z,p_\perp)\; \sin(\phi-\varphi_S)\, \sin(\varphi_h-\varphi_S)}{\sum_q e_q^2 \int d\varphi_S\, d\varphi_h\, d^2\vec{k}_\perp\; f_{q/p}(x,k_\perp)\, \frac{d\sigma}{dQ^2}\, D_{h/q}(z,p_\perp)}$$

Conclusions

- Shaping up the nucleon spin and $k_\perp$ structure

  (need more information on quark intrinsic motion)

- First ever extraction of the transverse distribution function

  (Simultaneus determination of the Collins fragmentation function through a global analysis of exp. data)

- Plenty of new data expected from JLab and BELLE

- New frontiers: Orbiting quarks ?

Cahn: the observed azimuthal dependence is related to the **intrinsic $k_{\perp}$** of quarks (at least for small P_T values)

$$k_{¿} = (k_{¿} \cos \phi, k_{¿} \sin \phi, 0)$$

$$\hat{s} = sx\left[1 - \frac{2\mathrm{k}_{¿}}{Q}\sqrt{1-y}\cos \phi\right] + O\left(\frac{k_{¿}^2}{Q^2}\right)$$

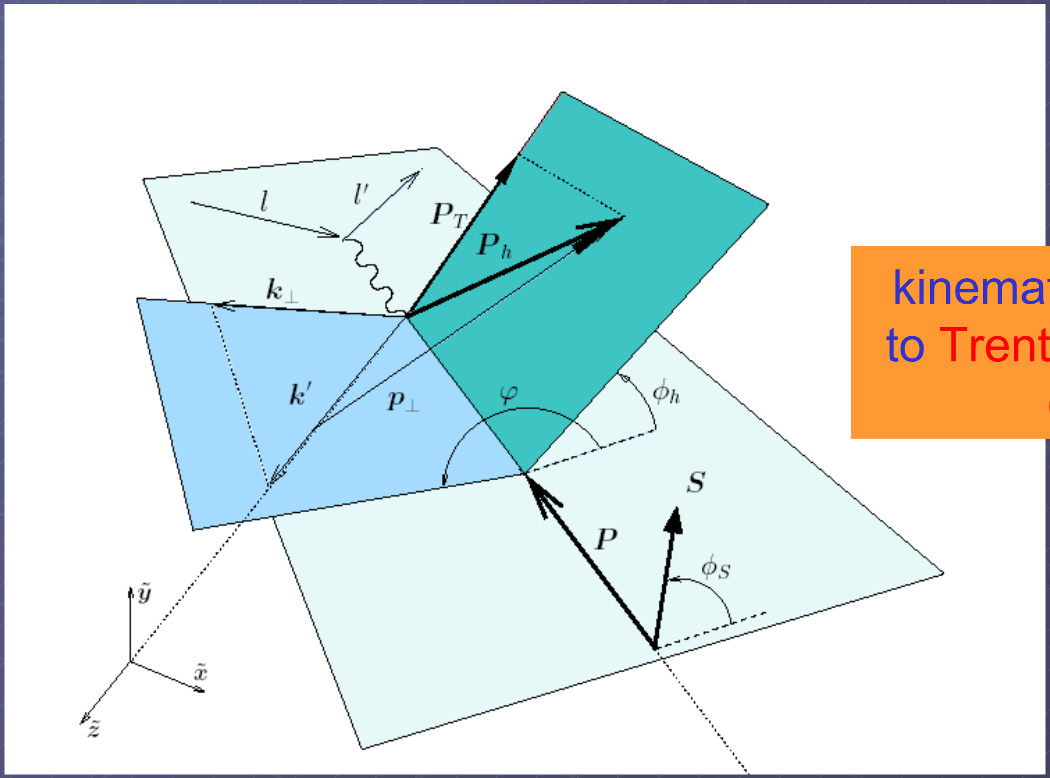
$$\hat{u} = s\,x\,(1-y)\left[1 - \frac{2\mathrm{k}_{¿}}{Q\sqrt{1-y}}\cos \phi\right] + O\left(\frac{k_{¿}^2}{Q^2}\right)$$

assuming collinear fragmentation, $\varphi = \Phi_h$

$$\frac{d\hat{\sigma}^{lq \to lhX}}{d\Phi_h} \propto \hat{s}^2 + \hat{u}^2 \propto A + B\cos\Phi_h + C\cos 2\Phi_h$$

These modulations of the cross section with azimuthal angle are denoted as "**Cahn effect**".

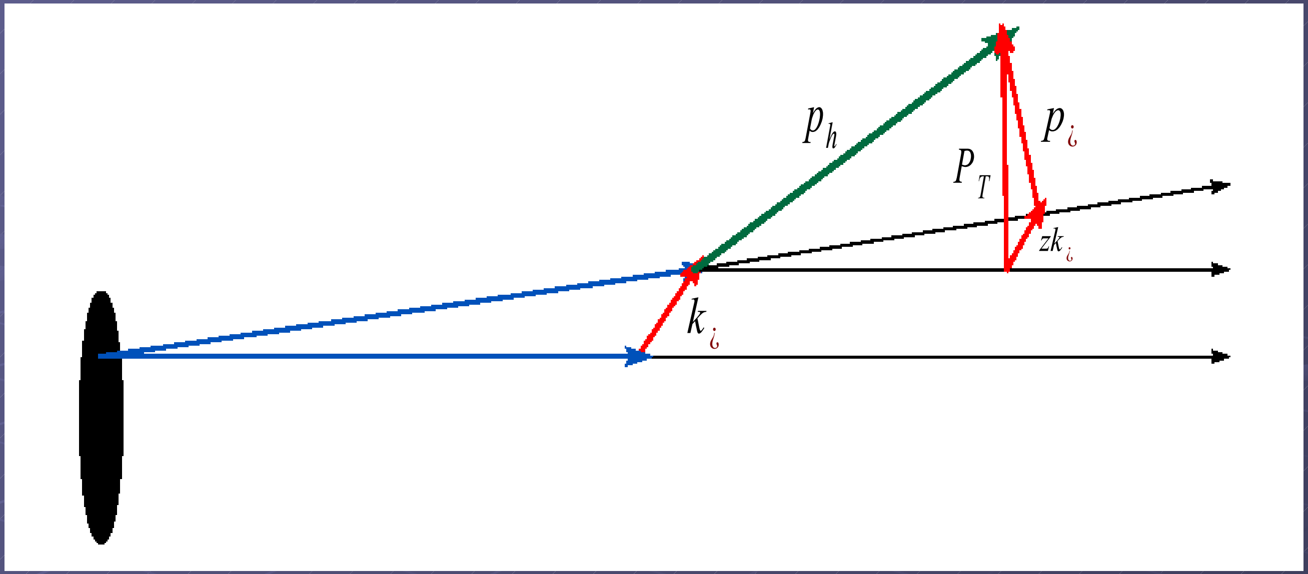
Sidis with intrinsic $k_{\perp}$



kinematics according to Trento conventions (2004)

Factorization holds at large Q^2, and $P_T \sim k_{\perp} \sim \lambda_{QCD}$ Ji, Ma, Yuan

$$d\sigma^{lp \to lhX} = \sum_q f_q(x, k_{\perp}; Q^2) \quad d\hat{\sigma}^{lq \to lq}(y, k_{\perp}; Q^2) \quad D_q^h(z, p_{\perp}; Q^2)$$



The full kinematics is complicated as the produced hadron has also intrinsic transverse momentum with respect to the fragmenting parton

Neglecting terms of order $(k_\perp^2 / Q^2)$ one has

$$P_T = p_\perp + zk_\perp$$

assuming:

$$f_q(x, k_\perp) = f_q(x)\,\frac{1}{\pi\langle k_\perp^2\rangle}\,e^{-k_\perp^2/\langle k_\perp^2\rangle}$$

$$D_q^h(z, p_\perp) = D_q^h(z)\,\frac{1}{\pi\langle p_\perp^2\rangle}\,e^{-p_\perp^2/\langle p_\perp^2\rangle}$$

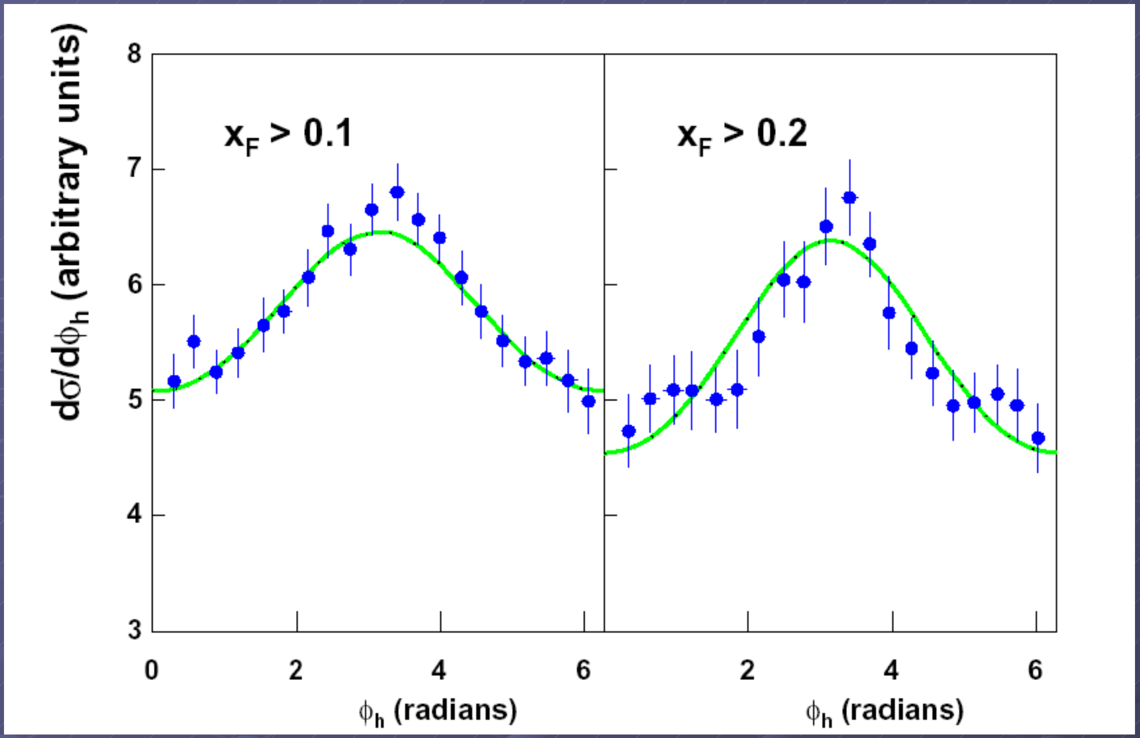
one finds:

$$\frac{d^5\sigma^{\ell p\to\ell h X}}{dx_B\,dQ^2\,dz_h\,d^2\boldsymbol{P}_T} \simeq \sum_q \frac{2\pi\alpha^2 e_q^2}{Q^4}\,f_q(x_B)\,D_q^h(z_h)\Big[1+(1-y)^2 - 4\,\frac{(2-y)\sqrt{1-y}\,\langle k_\perp^2\rangle\,z_h\,P_T}{\langle P_T^2\rangle\,Q}\,\cos\phi_h\Big]\frac{1}{\pi\langle P_T^2\rangle}\,e^{-P_T^2/\langle P_T^2\rangle}$$

with $\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle$ →

clear dependence on $\langle p_\perp^2\rangle$ and $\langle k_\perp^2\rangle$ (assumed to be constant)

Find best values by fitting data on Φ_h and P_T dependences



EMC data, μp and μd, *E* between 100 and 280 GeV

$$\langle k_\perp^2 \rangle = 0.28\ (\mathrm{GeV})^2 \qquad \langle p_\perp^2 \rangle = 0.25\ (\mathrm{GeV})^2$$

M.A., M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia and A. Prokudin