Random percolation and interquark confining potential

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Plan of the talk

- Lattice gauge theories and confinement:
  - String effects in confinement
- Random percolation as a gauge theory:
  - Definition and features of the model
  - Numerical confirmations
- Conclusions

Quark confinement in strong force

Still a puzzle after > 30 years and despite a number of approaches and ideas.

It is a million-dollar question, *literally* (Clay Mathematics Institute, 2000).

Lattice gauge theory (LGT) is the standard (nonperturbative) framework to study the problem.

To get rid of hadronisation: $\Rightarrow$ pure gauge theory (quark masses $\rightarrow \infty$).

The gauge fields U_ℓ (with group G) live on the (3+1)-D lattice links ℓ; the (Euclidean) *Wilson action* [Wilson 1974]:

$$Z = \sum_{\text{cfg.}} e^{-S} \; , \; S = \sum_{\square} \beta \left[\mathfrak{Re} \mathrm{Tr} U_\square \right] \;\; ; \;\; \beta \propto \frac{1}{g^2} \; , \; U_\square = U_{\ell_1} \cdots U_{\ell_4} \;\; .$$

Any G is allowed, even if discrete ($G = SU(3)$ gives QCD).

Observables

Gauge-invariant observables: ordered products of links along closed loops γ. In particular, with γ an R by T rectangle:

$$\langle W(R,T)\rangle = \left\langle \mathrm{Tr} \prod_{i \in \gamma} U_{\ell_i} \right\rangle \ , \ \text{Wilson loop}$$

All open lines are not gauge-invariant observables, since the gauge transformation is given by $U_\ell \to \Lambda(x)^\dagger U_\ell \Lambda(y)$, with $\ell = \langle x, y\rangle$.

Quark confinement on the lattice

The $R \times T$ loop represents a $q - \bar{q}$ couple at a distance R for a time T $\Rightarrow$ the interquark potential is

$$V(R) = - \lim_{T \to \infty} \frac{1}{T} \log \langle W(R,T) \rangle$$

- If $V(R) \sim \sigma \cdot R$, the system is confining ($\langle W(R,T) \rangle \propto \exp(\text{loop area})$)
- if $\sigma = 0$ there is deconfinement ($\langle W(R,T) \rangle \propto \exp(\text{loop perimeter})$)

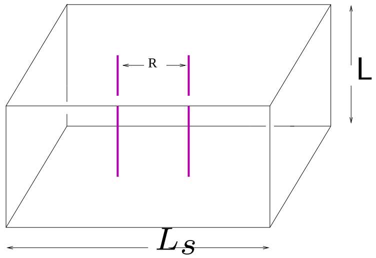
$\Rightarrow$ σ (string tension) is an order parameter.

Finite temperature and deconfinement

At **finite temperature** T, time becomes compact with length $L \propto 1/T$. W loses physical interest, and one reads σ from the *Polyakov loop correlator*:

$$\langle P(0) P^\dagger(R) \rangle_L \propto \exp(-\sigma R L + \cdots) \ .$$

Polyakov loop: a path γ nontrivially wound around the time direction.



At high enough $T > T_c(\beta)$, the system deconfines. If second-order transition:

- it behaves like a one-dimension-less spin model [Svetitsky, Yaffe 1982];
- universality of quantities such as $\frac{T_c}{\sqrt{\sigma}}$.

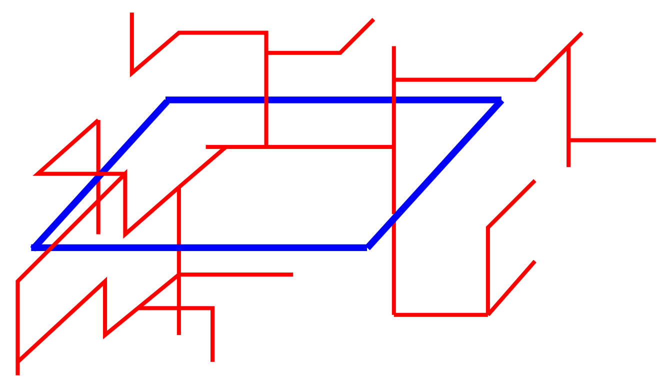
Actually, the above ratio (and other features) turn out to be almost G-independent: there must be a simple common confining mechanism.

Interpretations of confinement - I

In a given gauge configuration, there is a network of **center vortices**, extended objects related to "singular gauge transformations"; a vortex gives a multiplicative contribution to $W(R,T)$ if it pierces the loop, and there is confinement if the vortex graph is structured in such a way that $W(R,T)$ decays with an area law.

Supporting facts:

- Experimental observation of *center dominance*.
- Sensitivity of finite-temperature observables to the *center* of $\mathbb{G}$.

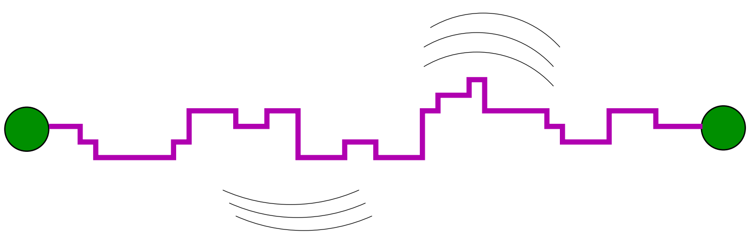


Interpretations of confinement - II

The vacuum acts on the (chromo-)electric charges as a **dual superconductor**, keeping all the flux between the sources squeezed in a string-like tube whose energy is proportional to its length $\Rightarrow$ linear growth of $V(R)$.

A strong coupling expansion of $\langle W \rangle$ can be made in terms of string worldsheet surfaces (bounded by the loop contour); this expansion fails for too weak couplings.

Indeed, since the physics takes place in the so-called rough phase (the string fluctuates quantistically on any length scale), there are many all-new predictions that can be tested on the lattice.



String-like properties on the lattice

Expectation value of a loop W as a sum over string worldsheets with border ∂W. The rough fluctuations give subleading quantum corrections to the area law, that can be tested on the lattice, e. g. (three dimensions):

$$W(R,R) \propto R^{1/4} e^{-\sigma R^2 - 2\mu R} \, .$$

The Nambu-Goto action $S \propto$ worldsheet area [Goto 1971; Nambu 1974], with due corrections [Polchinski, Strominger 1991; Hari Dass, Matlock 2006], leads to the *universality* of the first two orders in observables such as:

$$\langle P(0) P^\dagger(R) \rangle_{L=1/T} = \frac{e^{-cL - \sigma RL - \frac{(D-2)\pi^2 L [2E_4(\tau) - E_2^2(\tau)]}{1152 \sigma R^3} + \mathcal{O}(1/R^5)}}{\eta(\tau)^{D-2}} \; ; \; \tau \equiv \frac{iL}{2R} \, ,$$

and in scaling laws as:

$$\sigma(L) \stackrel{D=3}{=} \sigma - \frac{\pi}{6L^2} - \frac{\pi^2}{72 \sigma L^4} + \mathcal{O}(1/L^6) \qquad ; \quad L = \frac{1}{T} \, , \; R \to \infty \, .$$

For this to work, the theory is assumed to flow to a massless bosonic free string in the infrared limit $R \to \infty$.

From "simplest" to "even simpler"

Let's start with the 3D $\mathbb{Z}_2$ theory, the simplest nontrivial gauge model:

- Kramers-Wannier duality transformation $\Rightarrow$ 3D Ising model (one-to-one mapping between observables and couplings in the two models).
- Fortuin-Kasteleyn reformulation for the Ising model, in terms of random clusters of aligned sites.
- In this context, $W(R,T)$ is zero whenever a magnetised cluster is linked to the loop, since this fact is incompatible with the $\mathbb{Z}_2$ unit flux running on the loop contour.

Random percolation as a gauge theory

Naive, simple extension of the above recipe, in which only the topological properties are relevant: [Gliozzi, S. L., Panero, Rago: Nucl. Phys. B **719** (2005), 255]

- The lattice links are independently set to on or off according to probabilities p and $1-p$: a random cloud of connected on clusters is formed.
- This is a critical system: there exist a value p_{cr} at which an infinite connected network appears ($percolation\ threshold$, second order critical point).
- Formally, this model has $\mathbb{G} \equiv \{e\}$ and $Z \equiv 1$ (no update algorithm!).
- We need observables sensitive only to the configuration topology (no *dangling ends*, for instance), as some sort of gauge invariance.

Wilson loops and confinement in random percolation

One is led to define the Wilson loop value $W(\mathcal{C})$ in a given configuration $\mathcal{C}$ as:

$$W = 1 \text{ if no clusters are } \textit{linked} \text{ to the loop} \quad ; \quad W = 0 \text{ otherwise} \;\; .$$

The definition is completely invariant under any transformation that does not alter the *loop* structure of the configuration, as required.

$p < p_{cr} \Rightarrow$ finite-size clusters give the perimeter law $\Rightarrow$ deconfinement

$p > p_{cr} \Rightarrow$ the infinite cluster can pierce W in any point of $A \Rightarrow$ confinement:

$$p(0) = \binom{A}{0} \alpha^0 (1-\alpha)^{A-0} = \exp\Big\{ -\sigma A \Big\} \;\; \Rightarrow \;\; \sigma = -\log(1-\alpha)$$

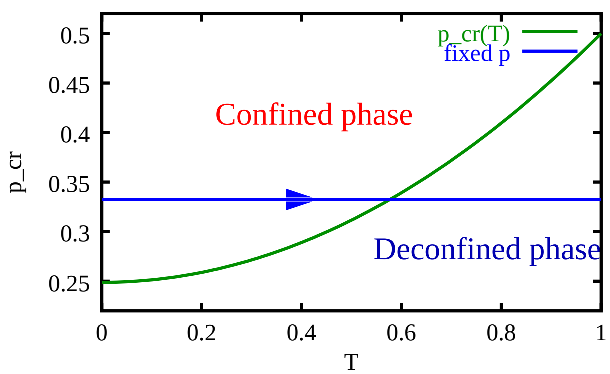
Finite-temperature deconfinement in random percolation

At finite temperature T, the system is a 2D infinite slice with thickness $\propto 1/T$: the threshold probability $p_{cr}(T)$ for the infinite cluster to appear is now a function of T and follows two-dimensional scaling laws.

Since $p_{cr}(T)$ is an increasing function of T, by keeping a fixed probability $p_{cr}(0) < p < p_{cr}(\infty)$ and heating up the system, the infinite cluster at some point T_{cr} vanishes away, leaving finite pieces that no longer give the area law to the loop behaviour: this is precisely a finite temperature deconfinement transition.

The amplitude ratio $T_{cr}/\sqrt{\sigma}$ is well defined, with scaling laws

$$\sigma(p) = S(p - p_{cr})^{2\nu}$$
$$T = \tau_0(p_{cr}(T) - p_{cr}^{3D})^{\nu}$$



String tension from Wilson loops

First, we measured the quantity $p_{cr}(T)$ for a variety of lattices and temperatures, using the Newman-Ziff algorithm.

From the expectation values of rectangular loops, a fit to an area+perimeter law can be tried, $\langle W(R,T)\rangle \propto exp[-\sigma RT - p(R+T)]$.

The agreement is, however, much better if one includes the Leading Order correction coming from the string rough fluctuations:

$$\langle W(R,T)\rangle \propto \sqrt{\frac{\eta(i)\sqrt{R}}{\eta(iT/R)}} \cdot exp[-\sigma RT - p(R+T)]$$

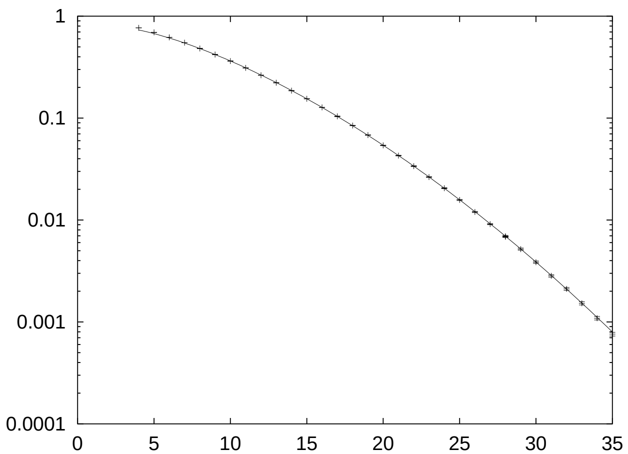
$\eta(\tau) = (e^{2i\pi\tau})^{1/24} \prod_{n=1}^{\infty} \left[1-(e^{2i\pi\tau})^n\right]$ is the Dedekind eta function.

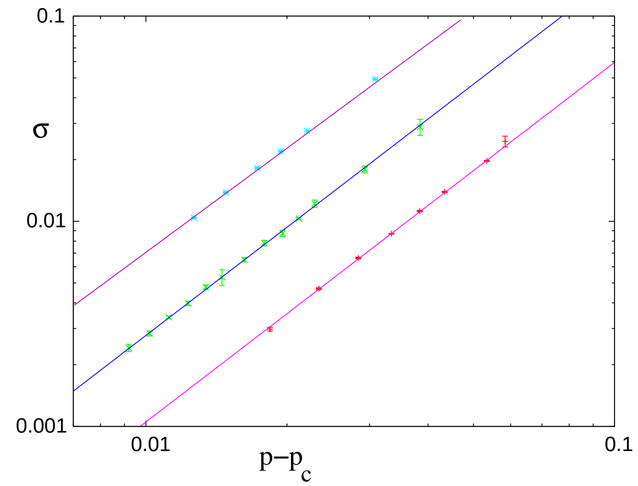
String tension from Wilson loops

The values obtained for σ show, not too far from $p_{cr}(0)$, a good scaling behaviour and allow to extrapolate the scaling amplitude S for each kind of lattice:

$$S_{\mathrm{SC,site}} = 3.370(8) \qquad S_{\mathrm{SC,bond}} = 8.90(3) \qquad S_{\mathrm{BCC,bond}} = 22.07(2)\ldots$$

The universal ratio $T_{cr}/\sqrt{\sigma} \simeq 1.5$ was calculated for seven different lattices and temperatures: its universality was proven within errors.





Polyakov-Polyakov correlators at critical temperature

Exactly at the critical point, the correlator between two Polyakov lines should exhibit a power-law shape, whose exponent is fixed by the dimensionality and universality class of the system.

Arguing that, at finite T, the system behaves according to 2D percolation universality class, one can use an adapted version of the Svetitsky-Yaffe conjecture to predict that:

$$\big\langle P(0)P(R)\big\rangle_{p=p_{cr}(T)} \propto R^{-\frac{5}{24}} \ .$$

From the measurement of such correlators, we could show that this expectation is fulfilled.

Pure gauge spectrum: glueballs

The plaquette-plaquette (zero-momentum projected) correlator shows a *multiple exponential* decay $\Rightarrow$ it couples to a whole tower of massive physical states in the 0^+ spin/parity channel.

Dihedral time-slice symmetry $\Rightarrow$ operators can be constructed for each channel with $J^P \in \{0^+, 0^-, 2^+, 2^-, 1/3\}$.

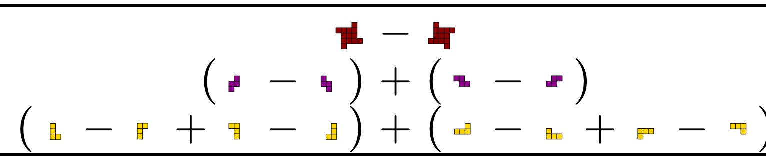
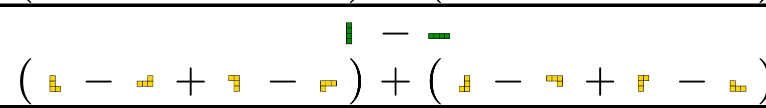
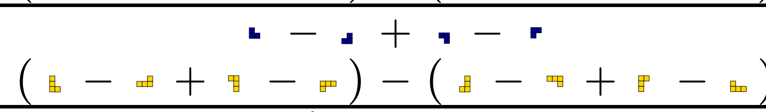
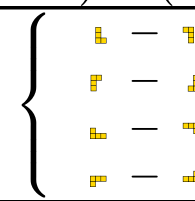
A cross-correlation matrix is constructed with:

$$\mathcal{C}_{ij}^{(J^P)}(t) = \sum_{x,y}^{(y-x)_3=t} \left[\langle \mathcal{O}_i^{(J^P)}(x) \mathcal{O}_i^{(J^P)}(y) \rangle - \langle \mathcal{O}_i^{(J^P)} \rangle \langle \mathcal{O}_j^{(J^P)} \rangle \right]$$

and then diagonalised with $\mathcal{C}(t > t_0)\overline{\mathbf{x}} = \lambda^{t_0}(t)\mathcal{C}(t_0)\overline{\mathbf{x}}$, to extract glueball masses from each channel.

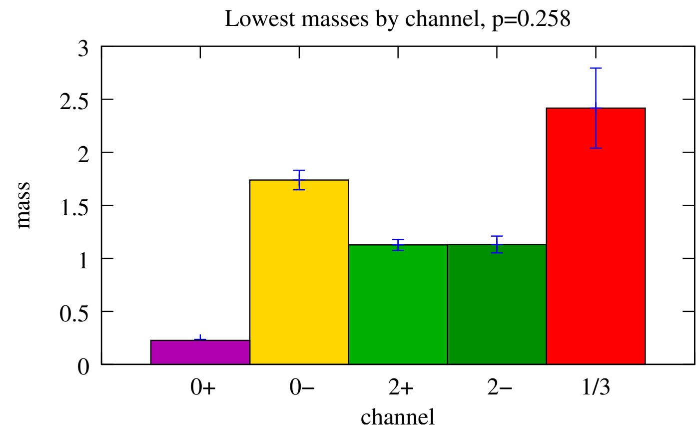
Glueballs operators' construction

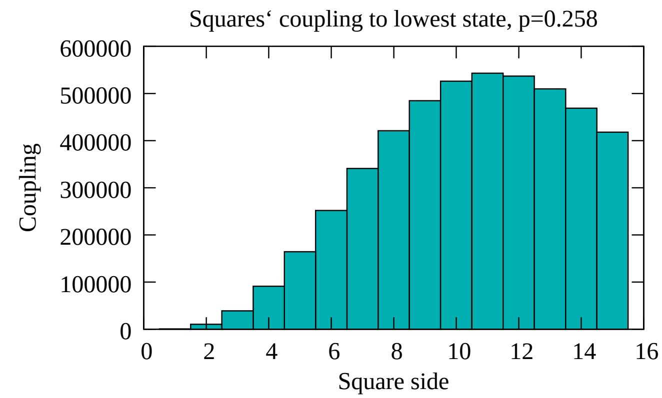
Choosing operators in various symmetry classes, we constructed spin/parity operators according to the dihedral character table:

0^-	
2^+	
2^-	
$1/3$	

Glueballs, results [Giudice, Gliozzi, S. L.: PoS(LATTICE 2007) 314 (2007)]

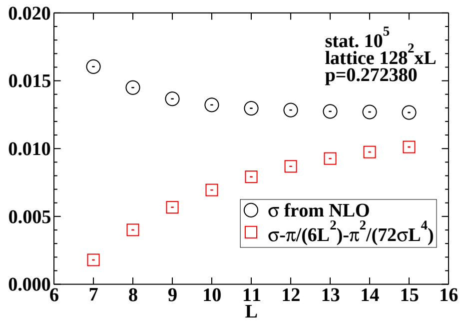
- The 0^+ lightest glueball shows good scaling.
- For each channel, the lowest state is easily recognizable: they follow the expected hierarchy, and $m_0^{0^+}/\sqrt{\sigma} \simeq 4.46$ (very close to the $SU(2)$ value 4.7).
- By looking for the square operator which maximises the coupling, we estimated the lowest scalar glueball size: its diameter turns out to be ~ 0.24 fm .





What is the underlying string theory?

Thanks to the high numerical precision attainable, from the Polyakov-Polyakov correlators the finite-temperature $\sigma(L)$ is extracted. [Giudice, Gliozzi, S. L.: **PoS**(LATTICE 2007) 314 (2007)]

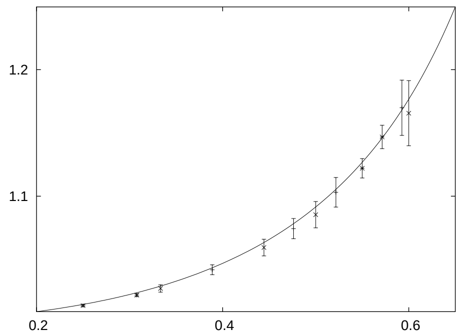


- Not only the $\langle PP \rangle$ is seen to follow the NLO prediction:

$$\sigma(L) = \underbrace{\sigma - \frac{\pi}{6L^2} - \frac{\pi^2}{72\sigma L^4}}_{\text{NLO}} + \underbrace{\frac{\pi^3}{C\sigma^2 L^6}}_{\text{non-universal}} + \mathcal{O}(1/L^8)\;;$$

- but in this system also the first model-dependent correction was clearly identified ($C \simeq 300$).

Universal functions as further signals of the rough phase



String evidences in the universal large-distance behaviour of:

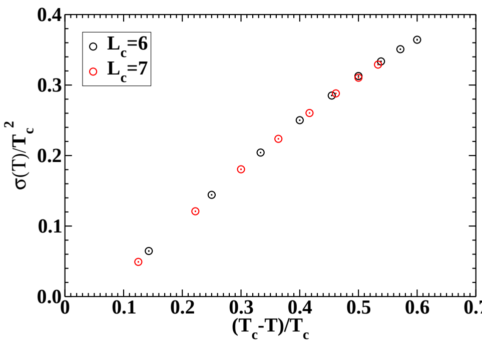
$$e^{n^2\sigma}\frac{W(R+n,R-n)}{W(R,R)} \;\to\; f(t)=\sqrt{\frac{\eta(i)\sqrt{1-t}}{\eta(i\frac{1+t}{1-t})}}\;,$$

with $t=\frac{n}{R}$.

At finite temperature, expected universality of the following ratio:

$$g(t)=\frac{\sigma(T)}{T_c^2}\;,$$

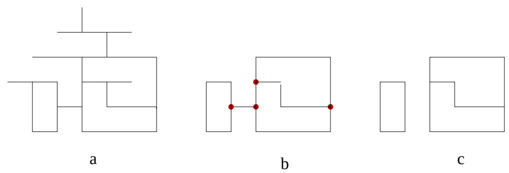
as a function of the reduced temperature $t=\frac{T_c-T}{T_c}$.



Some words on percolation algorithms . . .

All the measurements on the randomly-generated configurations involve looking for topological linking to some closed line.

Before taking measures, however, the configuration is mapped to its loop skeleton ("loop gauge") via removal of *dangling ends* and *bridges*. The cluster structure is constructed with the Hoshen-Kopelman algorithm: each node has a *parent node*, up to the cluster's root which points to itself.



To evaluate winding numbers, a ± 1 offset is associated to links dual to the loop surface in reconstructing clusters.

In the special case of the 1×1 plaquette, this can be avoided if we are in the "loop gauge".

A particularly optimised approach is implemented when (possibly a lot of) loops are to be measured in every spatial position.

Conclusions

- Percolation represents a well-defined gauge theory which retains all important features notwithstanding its simplicity.

- The model provides clear evidences of a fluctuating string behaviour. In fact, this is the only case in which the model-dependent features were identified, thus providing an actual realisation of a consistent string theory *à la* Nambu-Goto.

- The high numerical performance in the system played a key role in supporting the conjectures with extremely accurate "experimental" evidences.

Essential bibliography

Key references

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New results

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